Zero vector space contains subspace

That V is a subset of the $\R^n$ vector space that consists only of the zero vector of $\R^n$. That is, $V=\{\mathbf{0}\}$. Then prove that V is a subspace of $\R^n$. Add to later solve the issue of sponsored links content 292 Proof. What is the size of zero vector space? Proof. To prove that $V=\{\mathbf{0}\}$ is a subspace of $\R^n$, we check the following subspace criteria. Subspace Criteria (a) Vector zero $\mathbf{0} \in \R^n$ is in V. (b) If $\mathbf{x}, \mathbf{y} \in V$, then $\mathbf{x}+\mathbf{y}\in V$. (c) If $\mathbf{x} \in V$ and $c\in \R$, then $c\mathbf{x} \in V$. The condition (a) is clear, since V consists of the zero vector $\mathbf{0}$. To check condition (b), note that the only element in $V=\{\mathbf{0}\}$ is $\mathbf{0}$. Thus, if $\mathbf{x}, \mathbf{y} \in V$, then both $\mathbf{x}, \mathbf{y}$ are $\mathbf{0}$. Thus\ \[\mathbf{x}+\mathbf{y} =\mathbf{0}+\mathbf{0}=\mathbf{0}\in V\] and the condition (b) is met. To confirm the condition (c), leave $\mathbf{x}\in V$ and $c\in \R$. Next, $\mathbf{x}=\mathbf{0}$. We have \[c\mathbf{x}=c\mathbf{0}=\mathbf{0}\in V\] and the condition (c) is satisfied. Therefore, we checked all subspace criteria and therefore the subset $V=\{\mathbf{0}\}$ which consists only of vector zero is a subspace of $\R^n$. What is the size of zero vector space? What is the size of the subspace $V=\{\mathbf{0}\}$? The dimension of a subspace is the number of vectors on a base. So let's first find a V. Note that a V basis consists of vectors in V that are linearly independent scope sets. Because 0 is the only vector in V, the $S=\{\mathbf{0}\}$ set is the only possible set for a base. However, S is not a linearly independent set, since, for example, we have a nontrivial linear combination $1\cdot \mathbf{0}=\mathbf{0}$. Therefore, the subspace $V=\{\mathbf{0}\}$ has no basis. Therefore, the size of V is zero. Add to resolve links sponsored later Tags: basebasis for a spatial basis of a spacedimensionlinear vector algebralinear almost dependentlli almost independent combinations configuring setsubspatial vector criteria vector vectorzero vector vector space vector $\start group$ If this is true, then each vector space should always have at least one subspatial, the one that consists only of the zero vector, correct? Thank you! $\endgroup$ Basic algebraic structure of linear algebra should not be confused with the Vector field. This article is about linear (vector) spaces. For the structure in incidence geometry, see linear space (geometry). For the space technology company, see Vector Space Systems. Adding vectors and scalar multiplication: a vector v (blue) is added to another vector w (red, top illustration). Below, w is stretched by a factor of 2, yielding the sum v + 2w. Vector space (also called linear space) is a set of objects called vectors, which can be added and multiplied (scaled) by numbers, called scaling. Scalings are often taken to be real numbers, but there are also vector spaces with scalar multiplication by complex numbers, rational numbers or usually any field. Vector addition and scalar multiplication operations must meet certain requirements, called vector axiomas (listed below in § Definition). To specify that scaling scans are real or complex numbers, the terms actual vector space and complex vector space are often used. Certain sets of Euclidean vectors are common examples of a vector space. They represent physical quantities, such as forces, where any two forces (of the same type) can be added to produce a third, and multiplying a force vector by a real multiplier is another force vector. In the same vein (but in a more geometric sense), vectors representing plane displacements or three-dimensional space also form vector spaces. Vectors in vector spaces do not necessarily have to be arrow-like objects as they appear in the examples mentioned: vectors are considered abstract mathematical objects with particular properties, which in some cases can be viewed as arrows. Vector spaces are objects of linear algebra and are well characterized by their dimension, which roughly specifies the number of independent directions in space. Vector-dimensional spaces naturally arise in mathematical analysis as operating spaces, whose vectors are functions. These vector spaces are usually endowed with some additional structure, such as a topology, which allows consideration of proximity and continuity issues. Among these topologies, those that are defined by a standard or internal product are most commonly used (being equipped with a notion of distance between two vectors). This is particularly the case of the Banach and Hilbert spaces, which are fundamental in mathematical analysis. Historically, the first ideas that lead to vector spaces can be traced back to 17th-century analytical geometry, matrices, systems of linear equations, and Euclidean vectors. The modern and more abstract treatment, first formulated by Giuseppe Peano in 1888, encompasses more general objects than Euclidean space, but much of the theory can be seen as an extension of classical geometric ideas such as lines, planes and their upper-dimensional analogues. Today, vector vacancies are applied in all mathematics, science and engineering. They are the appropriate linear-algebraic notion for dealing with systems of linear equations. They provide a framework for fourier expansion, which is employed in image compression routines, and provide an environment that can be used for solution techniques for partial differential equations. In addition, vector spaces provide an abstract, coordinate-free form of dealing with geometric and physical objects, such as tensors. This, in turn, allows the examination of the local properties of varieties linearization techniques. Vector spaces can be generalized in several ways, leading to more advanced notions in geometry and abstract algebra. Algebraic Similar group Semigroup / Monoid Rack and quandle Quasigroup and loop Abelian group Magma Lie group Group theory Ring-like Ring Semiring Qua-ring Commutative Ring Integral domain Field division ring ring theory ritamento-like Latce Semilattice Lattice Order total Heyting algebra Boolean algebra Map of lattices Lates Theory Module-like Module-Like Module Group with Vector space operators Algebra-like Linear Algebra Algebra Association Nonassociative Nonassociative Composition Nonassociative Algebra Lie Algebra Graded Bialgebra vte Introduction and definition The concept of vector space will be explained first by describing two particular examples: First example: arrows in the plane The first example of a vector space consists of arrows on a fixed plane, starting at a fixed point. This is used in physics to describe forces or velocities. Given any of these arrows, v and w, the parallelogram covered by these two arrows contains a diagonal arrow that starts at the origin, too. This new arrow is called the sum of the two arrows, and is denoted v + w.[1] In the special case of two arrows on the same line, its sum is the arrow in this line whose length is the sum or the difference of lengths, depending on whether the arrows have the same direction. Another operation that can be done with arrows is sizing: given any positive actual number a, the arrow that has the same direction as v, but is dilated or shrunk by multiplying its length by a, is called multiplication of v by a. It is denoted av. When a is negative, av is set to the arrow pointing in the opposite direction instead. The following shows some examples: if a = 2, the resulting vector aw has the same direction as w, but is extended to the double length of w (image right below). Equivalently, 2w is the sum w +w. In addition, (−1)v = −v has the opposite direction and the same length of v (blue vector pointing down in the right image). Second example: pairs of ordered numbers A second key example of a vector space is provided by pairs of real numbers x and y. (The order of components x and y is significant, so such a pair is also called an ordered pair.) Such a pair is written as (x, y). The sum of two of these pairs and the multiplication of a pair with a number is defined as follows: (x 1 , y 1) + (x 2 , y 2) = (x 1 + x 2 , y 1 + y 2) {\displaystyle (x_{1} ,y_{1})+(x_{2},y_{2})=(x_{1}+x_{2},y_{1}+y_{2})} and a (x , y) = (a x , a y) {\displaystyle a(x,y)=(ax,ay)} . The first example above is reduced to this, if the arrows are represented by the pair of Cartesian coordinates of their endpoints. Definition In this article, vectors are represented in bold to distinguish them from scaling. [nb 1] A vector space over an F field is a Set V along with two operations that satisfy the eight axiomas listed below. Next, V × V denotes the Cartesian V product with itself, and → denotes a mapping from one set to another. The first operation, called adding or simply addition + : V × V → → takes any two vectors v and w and assigns them a third vector that is commonly written as v+w, and called the sum of those two vectors. (The resulting vector is also an element of set V.) The second operation, called scalar multiplication · : F × V → V, takes any scalar to and any vector v and gives another vector av. (Similarly, the av vector is an element of set V. Scalar multiplication should not be confused with the scalar product, also called an internal product or point product, which is an additional structure present in some specific spaces, but not all vector spaces. Scalar multiplication is a multiplication of a vector by a scalar; the other is a multiplication of two vectors producing one scalar.) V elements are commonly referred to as vectors. F elements are commonly called scaling. Common symbols for denoting vector spaces include U {\displaystyle U} , V {\displaystyle V} and W {\displaystyle W} . [1] In the two examples above, the field is the field of the actual numbers, and the set of vectors consists of the planar arrows with fixed starting point and pairs of real numbers, respectively. To qualify as vector space, the V-set and the addition and multiplication operations must adhere to a number of requirements called axiomas. [2] These are listed in the table below, where u, v, and w denote arbitrary vectors in V, and a and b denote scalar in F.[3][4] Axiom Meaning Associativity of addition u + (v + w) = (u + v) + w Commutativity of addition u + v = v + u Addition identity element There is an element 0 ∈ V, called vector zero, in such a way that v + 0 = v for all v ∈ V. Inverse elements of addition For each v ∈ V, there is an −v ∈ V, called the additive inverse of v, such that v + (−v) = 0. Compatibility of scalar multiplication with field multiplication a(bv) = (ab)v [nb 2] Element of identity of the scalar multiplication 1v = v, where 1 denotes the multiplicative identity in F. Distributivity of scalar multiplication in relation to vector addition a(u + v) = au + av Distributivity of scalar multiplication in relation to the addition of field (a + b)v = av + bv These axiomas generalize properties of the vectors introduced in the examples above. In fact, the result of adding two ordered pairs (as in the second example above) does not depend on the order of the summers: (xv, yv) + (xw, yw) = (xw, yw) + (xv, yv). Similarly, in the geometric example of vectors such as arrows, v + w = w + v, since the parallelogram that defines the sum of vectors is independent of the order of the vectors. All other axiomas can be checked similarly in both examples. Thus, disregarding the concrete nature of the particular type of vectors, the definition incorporates these two and many more examples into a notion of vector space. The subtraction of two vectors and the division by a scalding (non-zero) can be set to v − w = v + (− w) v a = 1 to v {\begin{alinhado}\mathbf {v} -\mathbf {w} &=\mathbf {v} +(-\mathbf {w} {w} {v} }{a}}&{{\frac {1}{a}}\mathbf {v} \end{aligned}}} . When the scalar field F is the actual number R, the vector space is called the actual vector space. When the scalar field is the complex number C, the vector space is called complex vector space. These two cases are the most used in engineering. The general definition of a vector space allows scalars to be elements of any fixed field F. The notion is then known as an F-vector space or a vector space over F. A field is essentially a set of numbers that have addition, subtraction, multiplication, and division operations. [nb 3] For example, rational numbers form a field. In contrast to the intuition resulting from vectors in the plane and cases of higher dimension, in general vector spaces, there is no notion of proximity, angles or distances. To deal with such issues, particular types of vector spaces are introduced; see § Vector spaces with additional structure below for more. Alternative formulations and elementary consequences Vector addition and scalar multiplication are operations, satisfying the closing property: u+v and av are in V for all a in F, and you, v in V. Some older sources mention these properties as separate axiomas. [5] In the language of abstract algebra, the first four axiomas are tantamount to requiring the vector set to be an abelian group under addition. The remaining axiomas give this group a module structure F. In other words, there is a f-ring homomorphism of the F field for the vector group endomorphism ring. Then, the scalar multiplication av is set to (f(a))(v). [6] There are a number of direct consequences of vector space axiomas. Some of them derive from the theory of the elementary group, applied to the additive group of vectors: for example, the vector zero 0 of V and the additive inverse −v of any vector v are unique. Other properties also continue to use distributive law for scalar multiplication, for example, av is equal to 0 if and only if an equal to 0 or v is equal to 0. History More information: History of algebra vector spaces comes from fine geometry, through the introduction of coordinates in the plane or three-dimensional space. Around 1636, the French mathematicians René Descartes and Pierre de Fermat founded analytical geometry by identifying solutions to an equation of two variables with points on a plane curve. [7] To achieve geometric solutions without the use of coordinates, Bolzano introduced, in 1804, certain operations on points, lines and airplanes, which are predecessors of vectors. [8] This work was done in the design of barycentric coordinates by Möbius in 1827. [9] The basis of the definition of vectors was Bellavitis's notion of the bipoint point, a oriented segment of one of which ends the origin and the other a target. The vectors were reconsidered with the presentation of complex numbers by Argand and Hamilton and the beginning of the by the latter. [10] They are elements in R2 and R4; treat them using linear linear back to Laguerre in 1867, which also defined systems of linear equations. In 1857, Cayley introduced matrix notation that allows for harmonization and simplification of linear maps. Around the same time, Grassmann studied the barycentric calculus initiated by Möbius. He predicted sets of abstract objects endowed with operations. [11] In his work, the concepts of independence and linear dimension are present, as well as the scaling products. In fact, Grassmann's work in 1844 exceeds the structure of vector spaces, since their multiplication also led to what are now called algebras. The Italian mathematician Peano was the first to give the modern definition of vector spaces and linear maps in 1888. [12] An important development of vector spaces is due to the construction of function spaces by Henri Lebesgue. This was later formalized by Banach and Hilbert around 1920. [13] At that time, algebra and the new field of functional analysis began to interact, not. [14] Also at this time, the first studies on vector spaces of infinite vectors were carried out. Examples Main article: Examples of vector spaces Coordinate space The simplest example of a vector space over an F field is the field itself, equipped with its standard addition and multiplication. More generally, all n-tuples (length sequences n) (a1, a2, ..., an) of F elements form a vector space that is usually denoted Fn and called coordinate space. [15] Case n = 1 is the simplest example mentioned above, in which field F is also considered as a vector space over itself. Case F = R and n = 2 was discussed in the introduction above. Complex numbers and other field extensions The set of complex numbers C, i.e. numbers that can be written in form x + iy for real numbers x and y where I am the imaginary unit, form a vector space over the reals with the usual addition and multiplication: (x + iy) + (a + ib) = (x + a) + i(y + b) and c ⋅ (x + iy) = (c ⋅ x) + i(c ⋅ y) for real numbers x, y, a, b, and c. The various axiomas of a vector space follow the fact that the same rules remain for complex inepic aritmetics. In fact, the example of complex numbers is essentially the same (that is, it is isomorphic) for the vector space of ordered pairs of real numbers mentioned above: if we think of the complex number x + iy as representing the ordered pair (x, y) in the complex plane, then we see that the rules of addition and multiplication scalar correspond exactly to those of the previous example. More generally, field extensions provide another class of examples of vector spaces, particularly in algebra and algebraic number theory: an F field containing a smaller field E is an E-vector space, by the given multiplication and addition operations of F. [16] For example, the complexes are a vector space over R, and the Field Extension Q (i 5) {\displaystyle \mathbf {Q} (i{\sqrt {5}})} is a space over Q. Main article functions: Function space Adding functions: The sum of the sine and exponential function is sin + exp : R → R {\displaystyle \sin +\exp \colon \mathbb {R} \to \mathbb {R} } with (sin + exp) (x) = sin (x) + exp (x) {\displaystyle (\sin +\exp)(x)=\sin(x)+\exp(x)} Functions of any fixed set Ω to an F field also form vector spaces, performing addition and multiplication scalar in point direction. That is, the sum of two functions f and g is the function (f + g) given by (f + g)(w) = f(w) + g(w), and similarly for multiplication. Such function spaces occur in many geometric situations, when Ω is the actual line or a range, or other subsets of R. Many topology and analysis such as continuity, integrability, or differentiability are well behaved in relation to linearity: sums and multiple scalar functions that have such property still have this property. [17] Therefore, the set of such functions are vector spaces. They are studied in more detail using functional analysis methods, see below. [clarification needed] Algebraic constraints also produce vector spaces: vector space F[x] is given by polynomial functions: f(x) = r0 + r1x + ... + rn−1xn−1 + rnxn, where the coefficients r0, ..., rn are in F.[18] Linear equations Main articles: Linear equation, linear differential equation and systems of linear equations Homogeneous linear equation systems are closely linked to vector spaces. [19] For example, solutions a + 3b + c = 0 4a + 2b + 2c = 0 are given by triples with a, arbitrary b = a/2 and c = −5a/2. They form a vector space: sums and multiples scalars of these triples still satisfy the same proportions of the three variables; so, are solutions, too. Arrays can be used to condense multiple linear equations above into a vector equation, that is, Ax = 0, where A = [1 3 1 4 2 2] {\displaystyle A={\begin{bmatrix}1&3&1\\4&2&2\end{bmatrix}}} is the matrix that contains the coefficients of the given equations, x is the vector (a, b, c), Ax denotes the matrix product, and 0 = (0, 0) is the zero vector. In a similar vein, solutions of homogeneous linear differential equations form vector spaces. For example, f''(x) + 2f'(x) + f(x) = 0 yields f(x) = ae−x + bxe−x, where a and b are arbitrary constants, and ex is the natural exponential function. Main articles base and dimension: Base and Dimension A vector v in R2 (blue) expressed in terms of different bases: using the default base of R2 v = xe1 + ye2 (black), and using a different, non-orthogonal base: v = f1 + f2 (red). Bases allow you to represent vectors by a sequence of scalars called coordinates or components. A base is a set (finite or infinite) B = {bi}i ∈ I of vectors bi, for convenience often indexed by some set of index I, which spans the entire space and is linearly independent. Covering the entire space means that any vector v can be expressed as a finite sum (called a linear combination) of the base elements: v = a 1 b i 1 + a 2 b i 2 + ⋯ + a n b i n , {\displaystyle \mathbf {v} =a_{1}\mathbf {b} _{i_{1}}+a_{2}\mathbf {b} _{i_{2}}+\cdots +a_{n}\mathbf {b} _{i_{n}},} (1) where ak is climbed, coordinates (or components) of vector v relative to base B, and bik (k = 1, ..., n) elements of B. Linear independence means that ak coordinates are determined exclusively for any vector in vector space. For example, the coordinate vectors e1 = (1, 0, ..., 0), and2 = (0, 1, 0, ..., 0), en = (0, 0, ..., 0, 1), form a base of Fn, called the default base, since any vector (x1, x2, ..., xn) can be expressed exclusively as a linear combination of these vectors: (x1, x2, ..., xn) = x1(1, 0, ..., 0) + x2(0, 1, 0, ..., 0) + ... + xn(0, ..., 0, 1) = x1e1 + x2e2 + ... + xnen. The corresponding coordinates x1, x2, ..., xn are just the Cartesian coordinates of the vector. Every vector space has a base. This comes from Zorn's motto, an equivalent formulation of the Axiom of Choice. [20] Given the other axioms of Zermelo-Fraenkel cluster theory, the existence of bases is equivalent to the axiom of choice. [21] The lemma ultrafilter, which is weaker than the axiom of choice, implies that all bases of a given vector space have the same number of elements, or cardinality (theorem of the dimension cf. for vector spaces). [22] It is called the dimension of vector space, denoted by dim V. If space is covered by many finite vectors, the above statements can be proven without such a fundamental contribution of set theory. [23] The dimension of the Fn coordinate space is n, by the base exposed above. The dimension of the polynomial ring F[x] introduced above[clarification needed] is countably infinite, a base is given by 1, x, x2, ... A fortiori, the dimension of more general function spaces, such as function space at some interval (delimited or not), is infinite. [nb 4] Under appropriate assumptions of regularity on the coefficients involved, the size of the solution space of a homogeneous ordinary differential equation is equal to the degree of the equation. [24] For example, the solution space for the above equation[clarification required] is generated by e−x and xe−x. These two functions are linearly independent over R, so the dimension of this space is two, as is the degree of the equation. A field extension over the rational Q can be considered as a vector space over Q (defining vector addition as field addition, defining scalar multiplication as field multiplication by Q elements, and ignoring field multiplication). The dimension (or degree) of the Q (α) field extension over Q depends on α. If α satisfies some polynomial equation q n α n + q n − 1 α n − 1 + ⋯ + q 0 = 0 {\displaystyle q_{n}\alpha ^{n}+q_{n-1}\alpha ^{n-1}+\ldots +q_{0}=0} with rational coefficients qn, ..., q0 (in other words, if α is algebraic), the dimension is finite. More precisely, it is equal to the degree of the minimum polynomial that has α as a root. [25] [25] For example, complex numbers C are a two-dimensional real vector space, generated by 1 and the imaginary unit i. The latter satisfies i2 + 1 = 0, a grade two equation. Thus, C is a two-dimensional vector space (and, like any field, one-dimensional as a vector space over itself, C). If α is not algebraic, the dimension of Q(α) over Q is infinite. For example, for α = π there is no such equation, in other words, π is transcendental. [26] Linear maps and matrices Main article: Linear map The relationship of two vector spaces can be expressed by linear map or linear transformation. These are functions that reflect the structure of vector space, i.e. preserve sums and scalar multiplications: f (v + w) = f (v) + f (w) {\displaystyle f(\mathbf {v} +\mathbf {w})=f(\mathbf {v})+f(\mathbf {w})} and f(a · v) = a · f(v) for all v and w in V, all in F.[27] An isomorphism is a linear map f : V → W such that there is an inverse map g : W → V, which is such a map that the two possible compositions f ∘ g : W → W and g ∘ f : V → V are identity maps. Equivalently, f is both one-to-one (injective) and over (surjective). [28] If there is an isomorphism between V and W, they say that the two spaces are isomorphic; they are then essentially identical as vector spaces, since all the identities they have in V are, via f, transported to similar in W, and vice versa via g. Describing an arrow vector v by its x and y coordinates produces an isomorphism of vector spaces. For example, the vector spaces "arrows in the plane" and ordered pairs of numbers in the introduction are isomorphic: a planar arrow v departing from the origin of some (fixed) coordinate system can be expressed as an ordered pair considering the x and y component of the arrow, as shown in the image on the right. On the other hand, given a pair (x, y), the arrow going by x to the right (or to the left, if x is negative), and y up (down, if y is negative) returns to arrow v. Linear maps V → W between two vector spaces form a vector space HomF(V, W), also denoted L(V, W). [29] The linear maps space from V to F is called double vector space, denoted V∗. [30] Through the natural injective map V → V∗∗, any vector space can be incorporated into its bidual; the map is an isomorphism if and only if space is finite-dimensional. [31] Once a V base is chosen, linear maps f : V → W are completely determined by specifying the images of the base vectors, because any element of V is expressed exclusively as a linear combination of them. [32] If dim V = dim W, a 1-1 match between fixed bases of V and W gives rise to a linear map that maps any base element of V to the corresponding base element of W. It is an isomorphism, by its own definition. [33] Therefore, two vector spaces are isomorphic if the two dimensions match and vice versa. Another way to express this is that any vector space is completely classified (up to isomorphism) by its dimension, a single number. Em any f-vector n-dimensional space V is isomorphic for Fn. There is, however, no canonical or preferred isomorphism; in fact, an isomorphism φ : Fn → V is equivalent to choosing a Base of V, mapping the default base from Fn to V, via φ. The freedom to choose a convenient basis is particularly useful in the infinite-dimensional context; see below. [clarification needed] Main Articles Matrices: Matrix and Determinant A typical array matrix is a useful notion for coding linear maps. [34] They are written as a rectangular array of scaling as in the image on the right. Any m-by-n matrix gives rise to a linear map from Fn to Fm, by the next x = (x 1 , x 2 , ⋯ , x n) ↦ (∑ j = 1 n a 1 j x j , ∑ j = 1 n a 2 j x j , ⋯ , ∑ j = 1 n a m j x j) , {\displaystyle \mathbf {x} =(x_{1},x_{2},\ldots ,x_{n})\mapsto \left(\sum _{j=1}^{n}a_{1j}x_{j},\sum _{j=1}^{n}a_{2j}x_{j},\ldots ,\sum _{j=1}^{n}a_{mj}x_{j}\right),} where ∑ {\displaystyle \sum } denotes sum, or using matrix A matrix multiplication with the coordinate vector x: x ↦ Ax. In addition, after choosing the bases of V and W, any linear map f : V → W is represented exclusively by an array through this assignment. [35] The volume of this parallel is the absolute value of the determinant of matrix 3 by 3 formed by vectors r1, r2 and r3. The determining point (A) of a square matrix A is a scalar that tells you whether the associated map is an isomorphism or not: being so is sufficient and necessary that the determinant is not. [36] The linear transformation of the NR corresponding to a true-by-n matrix to be orientation preserving if and only if its determinant is positive. Eigenvalues and eigenvectors Main article: Eigenvalues and eigenvectors Endomorphisms, linear maps f : V → V, are particularly important, because in this case vectors v can be compared with their image under f, f(v). Any satisfactory nonzero v vector λv = f(v), where λ is a scalar, is called eigenvector of f with eigenvalue λ.[nb 5][37] Equivalently, v is a kernel element of the difference f − λ · Id (where id is the V identity map → V). If V is finite-dimensional, this can be reformulated using determinants: f ter eigenvalue λ is equivalent to det(f − λ · Id) = 0. When spelling the determinant definition, the expression on the left side can be seen as a polynomial function in λ, called characteristic polynomial of f.[38] If the F field is large enough to contain a zero of this polynomial (which happens automatically for algebraically closed F, such as F = C) any linear map has at least one eigenvector. Vector space V may or may not have an eigenbase, a base composed of eigenvectors. This phenomenon is governed by Jordan's canonical form of the map. [39] [nb 6] The set of all eigenvectors corresponding to a given eigenvalue of f forms a vector space known as eigenspace corresponding to the eigenvalue (and f) in question. To the spectral theorem, the statement in the infinite-dimensional case, the machinery of functional analysis is necessary, see below. [clarification needed] Basic constructions In addition to the concrete examples above, there are a number of standard linear algebraic constructions that produce vector spaces related to the data. In addition to the definitions below, they are also characterized by universal properties, which determine an X object by specifying linear maps of X for any other vector space. Subspaces and quotient spaces Main article: Linear subspatial vector space and quotient A line that passes through the origin (blue, thick) in R3 is a linear subspace. It is the intersection of two planes (green and yellow). A nonempty W subset of a Vector Space V that is closed under scalar addition and multiplication (and therefore contains the 0-vector of V) is called a linear subspace of V, or simply a subspace of V, when ambient space is unequivocally a vector space. [nb 7] V subspaces are vector spaces (over the same field) by themselves. The intersection of all subspaces containing a given set S of vectors is called their extent, and it is the smallest subspace of V containing the set S. Expressed in terms of elements, the span is the subspace composed of all linear combinations of Elements of S.[41] A linear subspace of dimension 1 is a vector line. A linear subspace of dimension 2 is a vector plane. A linear subspace that contains all elements except one of an ambient space is a vector hyperplane. In a finite-dimension vector space n, a vector hyperplane is therefore a subspace of dimension n -1. The counterpart to subspaces are quotient vector spaces. [42] Given any subspace W ⊂ V, the quotient space V/W of V (W modulo) is defined as follows: as a set, it consists of v + W = {v + w : w ∈ W}, where v is an arbitrary vector in V. The sum of two of these elements v1 + W and v2 + W is (v1 + v2) + W, and scalar multiplication is given by a · (v + W) = (a · v) + W. The key point in this definition is that v1 + W = v2 + W if and only if the difference of v1 and v2 is in W.[nb 8] This way, the quotient space V/W forgets the information that is contained in the W sub → space. [44] The existence of nuclei and images is part of the statement that the category of vector spaces (on a fixed field F) is an abelian category, that is, a corpus of mathematical objects and maps of preservation of structures between them (category) that behaves much like the category of abelian groups. [45] Because of this, many statements such as the first isomorphism theorem (also called the classification-nullity theorem in matrix terms) V/ker(f) ≡ im(f). and the theorem of the second and third isomorphism can be formulated and proven very similarly to the corresponding statements for groups. An important example is the core of a linear map x ↦ Ax for some fixed matrix A, as above. [clarification needed] The core of this map is the subspace of vectors x such that Ax = 0, which is precisely the set of solutions for the system of homogeneous linear equations belonging to A. This concept also extends to linear differential equations a 0 f + a 1 d f d x + a 2 d 2 f d x 2 + ⋯ + a n d n f d x n = 0 {\displaystyle a_{0}f+a_{1}{\frac {df}{dx}}+a_{2}{\frac {d^{2}f}{dx^{2}}}+\cdots +a_{n}{\frac {d^{n}f}{dx^{n}}}=0} , where ai coefficients are functions in x, too. In the corresponding map f ↦ D (f) = ∑ i = 0 n a i d i f d x i , {\displaystyle f\mapsto D(f)=\sum _{i=0}^{n}a_{i}{\frac {d^{i}f}{dx^{i}}},} the derivatives of the function appear linearly (as opposed to f''(x)2, for example). As differentiation is a linear procedure (i.e., (f + g)′ = f′ + g ′ and (c·f)′ = c·f′ for a constant c) this assignment is linear, called the linear differential operator. In particular, the solutions for the differential equation D(f) = 0 form a vector space (over R or C). Direct product and direct sum Main articles: Direct product and direct sum of modules The direct product of vector spaces and the direct sum of vector spaces are two ways to combine an indexed family of vector spaces into a new vector space. The direct product ∏ i ∈ I V i {\displaystyle \textstyle {\prod _{i\in I}V_{i}}} of a family of vector spaces Vi consists of the set of all tuples (vi)i ∈ I, which specify for each index i in some index set I an element vi of Vi. [46] The addition and multiplication scalar is performed in a component way. A variant of this construct is the direct sum ⊕ i ∈ I V i {\displaystyle \textstyle {\bigoplus _{i\in I}V_{i}}} (also called a co-product and denoted ∐ i ∈ I V i {\displaystyle \textstyle {\coprod _{i\in I}V_{i}}}), where only tuples with non-zero vectors are allowed. If the index set I is finite, the two buildings agree, but in general are different. Tensioning product Main article: Tensor product The tensor product V ⊗F W, or simply V ⊗ W, of two vector spaces V and W is one of the central notions of multilinear algebra that deals with extension notions such as linear maps to various variables. A g map : V × W → X is called bilinear if g is linear in both variables v and w. That is, for fixed w the map v ↦ g(v, w) is linear in the sense above and similarly to fixed v. The tensor product is a particular vector space that is a universal recipient of bilinear maps g, as follows. It is defined as the vector space consisting of finite (formal) sums of symbols called tensioners v1 ⊗ w1 + v2 ⊗ w2 + ⋯ + vn ⊗ wn, subject to the rules to · (v ⊗ w) = (a · v) ⊗ w = v ⊗ (a · w), where a is a scalar, (v1 + v2) ⊗ w = v1 ⊗ w + v2 ⊗ w, and v ⊗ (w1 + w2) = v ⊗ w1 + v ⊗ w2. [47] Commutative diagram representing the universal property of the tensor product. These rules ensure that the f map of the V to V ⊗ W that maps a tuple (v, w) to v ⊗ w is bilinear. O O states that given any vector space X and any bilinear map g : V × W → X, there is a unique g map u, shown in the diagram with a dotted arrow, whose composition with f is equal to g: u(v ⊗ w) = g(v, w).[48] This is called the universal property of the tensor product, an instance of the method—widely used in advanced abstract algebra—to indirectly define objects specifying maps to or from this object. Vector spaces with additional structure From the point of view of linear algebra, vector spaces are completely understood to the extent that any vector space is characterized, up to isomorphism, by its dimension. However, the vector spaces themselves do not provide a framework for dealing with the question—crucial to the analysis—of whether a sequence of functions converges to another function. Similarly, linear algebra is not adapted to handle infinite series, since the addition operation allows only finitely many terms to be added. Therefore, functional analysis needs require additional structures to be considered. A vector space can receive a partial order ≤, under which some vectors can be compared. [49] For example, the real n-dimensional space Rn can be ordered by comparing its vectors in a component way. Ordered vector spaces, for example, riesz spaces, are fundamental for lebesgue integration, which is based on the ability to express a function as a difference of two positive functions f = f+ − f−, where f+ denotes the positive part of f and f− the negative part. [50] Standard vector spaces and internal product spaces Main articles: Normed vector space and internal product space Measuring vectors is made by specifying a standard, a datum that measures vector lengths, or by an internal product, that measure angles between vectors. Internal standards and products are denoted | v | {\displaystyle |\mathbf {v} |} and ⟨ v , w ⟩ {\displaystyle \langle \mathbf {v} ,\mathbf {w} \rangle } , respectively. The datum of an internal product implies that the lengths of the vectors can also be defined by defining the associated standard | v | := ⟨ v , v ⟩ {\textstyle |\mathbf {v} |:={\sqrt {\langle \mathbf {v} ,\mathbf {v} \rangle }}} . The vector spaces endowed with this data are known as standard vector spaces and internal product spaces, respectively. [51] Fn coordinate space can be equipped with the standard point product: ⟨ x , y ⟩ = x ⋅ y = x 1 y 1 + ⋯ + x n y n . {\displaystyle \langle \mathbf {x} ,\mathbf {y} \rangle =\mathbf {x} \cdot \mathbf {y} =x_{1}y_{1}+\cdots +x_{n}y_{n}.} [52] In R2, this reflects the common notion of the angle between two x and y vectors, by the law of cosines: x ⋅ y = cos (∠ (x , y)) ⋅ | x | ⋅ | y | . {\displaystyle \mathbf {x} \cdot \mathbf {y} =\cos \left(\angle (\mathbf {x} ,\mathbf {y})\right)\cdot |\mathbf {x} |\cdot |\mathbf {y} |.} Because of this, two vectors satisfying ⟨ x , y ⟩ = 0 {\displaystyle \langle \mathbf {x} ,\mathbf {y} \rangle =0} are called orthogonal. An important variant of the standard point product is used in the Minkowski space: R4 endowed with the Lorentz ⟨ x | y ⟩ = x 1 y 1 + x 2 y 2 + x 3 y 3 − x 4 y 4 . {\displaystyle \langle \mathbf {x} |\mathbf {y} \rangle =x_{1}y_{1}+x_{2}y_{2}+x_{3}y_{3}-x_{4}y_{4}.} [53] In contrast to the standard point product, it is not definitive positive: ⟨ x | x ⟩ {\displaystyle \langle \mathbf {x} |\mathbf {x} \rangle } also takes negative values, for example for x = (0 , 0 , 0 , 1) {\displaystyle \mathbf {x} =(0,0,0,1)} . Singling the fourth coordinate — corresponding to time, as opposed to three spatial dimensions — makes it useful for the mathematical treatment of special relativity. Topological Vector Spaces Main article: Topological vector space Convergence issues are addressed considering vector spaces V carrying a compatible topology, a structure that allows one to talk about elements close to each other. [54] Compatible here means that scalar addition and multiplication must be continuous maps. More or less, if x and y in V, and a in F vary by a limited amount, then so does x + y and ax.[nb 9] To make sense to specify the amount of a scaling changes, field F also has to load a topology in this context; a common choice are the real or the complex numbers. In these topological vector spaces one can consider series of vectors. The infinite sum ∑ i = 0 ∞ f i {\displaystyle \sum _{i=0}^{\infty }f_{i}} denotes the limit of the corresponding finite partial sums of the (fi)i∈N sequence of V elements. For example, fi can be functions (real or complex) belonging to some function space V, in which case the series is a series of functions. The convergence mode of the series depends on the topology imposed on the function space. In such cases, point convergence and uniform convergence are two prominent examples. The unit spheres in R2 consist of plane vectors of standard 1. The unit spheres are portrayed in different p-norms, for p = 1, 2 and ∞. The larger diamond depicts points of 1-norm equal to 2. One way to ensure the existence of limits of certain infinite series is to restrict attention to spaces where any Cauchy sequence has a boundary; such a vector space is called complete. Approximately, a vector space is complete as long as it contains all the necessary boundaries. For example, the vector space of polynomials in the unit range [0,1], equipped with the uniform convergence topology is not complete because any continuous function in [0,1] can be evenly approximated by a sequence of polynomials, by the Weierstrass approximation theorem. [55] In contrast, the space of all continuous functions in [0,1] with the same topology is complete. [56] A standard gives rise to a topology, defining that a sequence of vectors vn converges to v if and only if lim n → ∞ | v n − v | = 0. {\displaystyle {\text{lim}}_{n\rightarrow \infty }|\mathbf {v} _{n}-\mathbf {v} |=0.} The Banach and Hilbert spaces are complete topological vector spaces whose topologies are given, respectively, by a standard and an internal product. This study — a key piece of functional analysis — focuses on infinite dimension spaces, since all norms on topological vector spaces of finite dimension give rise to the same notion of convergence. [57] The image on the right shows the equivalence of standard 1 and ∞-standard in R2: as the balls unit close, a sequence converges to zero in one standard if and only if it happens in the other standard. In the infinite-dimensional case, however, there will usually be inequivalent topologies, which makes the study of topological vector spaces richer than that of vector spaces without additional data. From a conceptual point of view, all the notions related to topological vector spaces must match the topology. For example, instead of considering all linear maps (also called functional) V → W, maps between topological vector spaces are required to be continuous. [58] In particular, the (topological) double space V∗ consists of continuous functional V → R (or for C). Hahn-Banach's fundamental theorem is concerned with separating spaces subspaces from topological vector spaces appropriate by continuous functional. [59] Banach spaces Main article: Banach Space Banach spaces, introduced by Stefan Banach, are complete vector spaces. [60] A first example is vector space ℓ p {\displaystyle \ell ^{p}} composed of infinite vectors with real inputs x = (x 1 , x 2 , … , x n , …) {\displaystyle \mathbf {x} =\left(x_{1},x_{2},\ldots ,x_{n},\ldots \right)} whose p-norm (1 ≤ p ≤ ∞) given by ‖ x ‖ p := (∑ i | x i | p) 1 / p {\displaystyle \left\Vert \mathbf {x} \right\Vert _{p}:=\left(\sum _{i}\left\vert x_{i}\right\vert ^{p}\right)^{\frac {1}{p}}} for p < ∞ and ‖ x ‖ ∞ := sup i | x i | {\displaystyle \left\Vert \mathbf {x} \right\Vert _{\infty }:={\text{sup}}_{i}\left\vert x_{i}\right\vert } . Topologies in infinite-dimensional space ℓ p {\displaystyle \ell ^{p}} are inequivalent to different p. For example, the vector sequence x n = (2 − n , 2 − n , … , 2 − n , 0 , 0 , …) {\displaystyle \mathbf {x} _{n}=\left(2^{-n},2^{-n},\ldots ,2^{-n},0,0,\ldots \right)} The components are 2 − n {\displaystyle 2^{-n}} and the following are 0 {\displaystyle 0} , converges to vector zero to p = ∞ {\displaystyle p=\infty } , but not to p = 1 {\displaystyle p=1} : ‖ x n ‖ ∞ = sup (2 − n , 0) = 2 − n → 0 {\displaystyle \left\Vert \mathbf {x} _{n}\right\Vert _{\infty }=\sup(2^{-n},0)=2^{-n}\rightarrow 0} , but ‖ x n ‖ 1 = ∑ i = 1 2 n 2 − n = 2 n ⋅ 2 − n = 1. {\displaystyle \left\Vert \mathbf {x} _{n}\right\Vert _{1}=\sum _{i=1}^{2^{n}}2^{-n}=2^{n}\cdot 2^{-n}=1.} More generally than sequences of real numbers, the functions f : Ω → R {\displaystyle f\colon \Omega \to \mathbb {R} } are endorsed with a standard that replaces the sum above with the sum integral lebesgue ‖ f ‖ p := (∫ Ω | f (x) | p d μ (x)) 1 p . {\displaystyle \left\Vert {f}\right\Vert _{p}:=\left(\int _{\Omega }\left\vert {f}\left(x\right)\right\vert ^{p}\,{d\mu \left(x\right)}\right)^{\frac {1}{p}}.} The integral function space in a given domain is Ω {\displaystyle \Omega } (for example, a range) satisfying ‖ f ‖ p< ∞ {\displaystyle \left\Vert {f}\right\Vert _{p}<\infty } , and equipped with this standard are called Lebesgue spaces, denoted L p (Ω) {\displaystyle L^{\;\!p}\left(\Omega \right)} . [nb 10] These spaces are complete. [61] (If someone uses the Riemann integral instead, the space is not complete, which can be seen as a justification for Lebesgue's theory of integration.[nb 11]) Concretely this means that for any sequence of lebesgue-integrable functions f 1 , f 2 , ... , f n ... {\displaystyle f_{1},f_{2},\ldots ,f_{n},\ldots } with ‖ f n ‖ p< ∞ {\displaystyle \left\Vert {f}_{n}\right\Vert _{p}<\infty } , satisfying the condition lim k , n → ∞ ∫ Ω | f k (x) − f n (x) | p d μ (x) = 0 {\displaystyle \lim _{k,\ n\to \infty }\int _{\Omega }\left\vert {f}_{k}(x)-{f}_{n}(x)\right\vert ^{p}\,{d\mu \left(x\right)}=0} there is a function f (x) {\displaystyle {f}\left(x\right)} belonging to vector space L p (Ω) {\displaystyle L^{\;\!p}\left(\Omega \right)} in such a way that lim k → ∞ ∫ Ω | f (x) − f k (x) | p d μ (x) = 0. {\displaystyle \lim _{k\to \infty }\int _{\Omega }\left\vert {f}\left(x\right)-{f}_{k}\left(x\right)\right\vert ^{p}\,{d\mu \left(x\right)}=0.} Imposing delimiting conditions not only on the function, but also on its derivatives leads to Sobolev spaces. [62] Spaces Hilbert Main article: Space Hilbert The snapshots that succeed show sum of 1 to 5 terms in the approximation of a periodic function (blue) by finite sum of sine functions (red). Complete spaces of the internal product are known as Hilbert spaces, named after David Hilbert. [63] The Hilbert L2(Ω) space, with internal product given by ⟨ f , g ⟩ = ∫ Ω f (x) g (x) d x , {\displaystyle \langle f\ , g\rangle =\int _{\Omega }f(x){\overline {g(x)}}\,dx,} where g (x) {\displaystyle {\overline {g(x)}}} denotes the complex conjugate of g(x),[64][nb 12] is a key case. By definition, in a Hilbert space any Cauchy sequence converges to a boundary. On the other hand, finding a sequence of fn functions with desirable properties that approximates a particular boundary function is equally crucial. The initial analysis, under the pretext of Taylor's approach, established an approximation of different functions f by polynomies. [65] By the Stone-Weierstrass theorem, each continuous function in [a, b] can be approximated as closely as desired by a polynomial. [66] A similar approximation technique by trigonometric functions is commonly called Fourier expansion, and is widely applied in engineering, see below. [clarification needed] More generally, and more conceptually, the theorem produces a simple description of what basic functions, or, in Hilbert's abstract spaces, which are sufficient to generate a Hilbert H space, in the sense that the closing of its span (i.e., finite linear combinations and limits of it's all space. Such a set of functions is called the Base of H, its cardinality is known as the hilbert spatial dimension. [nb 13] Not only does the theorem display adequate base functions as sufficient for approximation purposes, but also together with the Gram-Schmidt process, it allows to build a base of orthogonal vectors. [67] Such orthogonal bases are the Hilbert spatial generalization of coordinate axes in finite-dimensional Euclidean space. Solutions to various differential equations can be interpreted in terms of Hilbert spaces. For example, many fields in physics and engineering lead to such equations, and often solutions with particular physical properties are used as basic functions, often orthogonal. [68] As an example of physics, the time-dependent Schrödinger equation in quantum mechanics describes the change of physical properties in time through a partial differential equation, whose solutions are called wave functions. [69] Definitive values for physical properties, such as energy, or impulse, correspond to eigen values of a given differential (linear) operator, and the associated wave functions are called eigenstates. The spectral theorem decomposes a compact linear operator acting in functions in terms of these eigen functions and their eigen values. [70] Algebras on fields Main articles: Algebra on a field and algebra lie A hyperbola, given by equation x · y = 1. The coordinate ring of the functions in this hyperbola is given by R[x, y] / (x · y − 1), a vector space of infinite dimension over R. General vector spaces do not have a multiplication between vectors. A vector space equipped with an additional bilinear operator defining the multiplication of two vectors is an algebra over a field. [71] Many algebras stem from functions in some geometric object: since functions with values in a given field can be multiplied in meaning, these entities form algebras. The Stone-Weierstrass theorem, for example, has banach algebras that are Banach spaces and algebras. Commutative algebra makes great use of polynomies rings in one or more variables, introduced above. [clarification needed] Its multiplication is at the same time commutative and associative. These rings and their quotients form the basis of algebraic geometry, because they are function rings of algebraic geometric objects. [72] Another crucial example is Lie algebras, which are neither commutative nor associative, but the failure to be so is limited by the restrictions ([x, y] denotes the product of x and y): [x, y] = −[y, x] (anticommutativeness), and [x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0 (Jacobi identities). [73] Examples include the vector space of n-by-n matrices, with [x, y] = xy − yx, the two-matrice switch, and R3, equipped with the cross-product. T(V) tensor algebra is a formal way to add products to any Vector Space V for algebra. [74] As a vector, it is crossed by symbols, called simple v1 ⊗ v2 ⊗ ⋯ ⊗ vn, where grade n varies. Multiplication is given by the concatenation of these symbols, imposing distributive law under addition, and requiring that multiplication escalate with the tensor product ⊗, in the same way as with the tensor product of two vector spaces introduced above. [clarification needed] In general, there are no relationships between v1 ⊗ v2 and v2 ⊗ v1. Forcing two of these elements to be equal leads to symmetric algebra, while forcing v1 ⊗ v2 = − v2 ⊗ v1 yields outer algebra. [75] When a field, F is explicitly declared, a common term used is F-algebra. Applications Vector spaces have many applications because they often occur in common circumstances, that is, wherever functions with values in some field. They provide a framework for dealing with analytical and geometric problems, or are used in the Fourier transformation. This list is not exhaustive: there are many more applications, for example, in optimization. The minimax theorem of game theory stating the existence of a one-time payout when all players play optimally can be formulated and proven using vector space methods. [76] The theory of representation fruitfully transfers good understanding of linear algebra and vector spaces to other mathematical domains, such as group theory. [77] Distributions Main article: Distribution Distribution (or generalized function) is a linear map assigning a number to each test function, typically a smooth function with compact support, continuously: in the terminology above[clarification required] the distribution space is the dual (continuous) of the test function space. [78] This latter space is endowed with a topology that takes into account not only f itself, but also all its higher derivatives. A standard example is the result of integrating an f test function over some domain Ω: I (f) = ∫ Ω f (x) d x . {\displaystyle I(f)=\int _{\Omega }f(x)\,dx.} When Ω = {p}, the set consisting of a single point, this reduces to the dirac distribution, denoted by δ, which associates a test function f its value in p: δ(f) = f(p). Distributions are a powerful tool for solving differential equations. Because all standard analytical nodes, as derived, are linear, they naturally extend to the space of distributions. Therefore, the equation in question can be transferred to a distribution space, which is larger than the underlying function space, so that more flexible methods are available to solve the equation. For example, Green's fundamental functions and solutions are usually distributions rather than appropriate functions, and can then be used to find equation solutions with prescribed boundary conditions. The solution found can, in some cases, be proven to be a true function, and a solution to the original equation (e.g., using the Lax-Milgram theorem, a consequence of the riesz's representation). [79] Fourier Analysis Main Article: Fourier Analysis The Heat Heat Equation the dissipation of physical properties over time, such as the decline in the temperature of a warm body placed in a colder environment (yellow portrays cooler than red regions). Solving a periodic function in a sum of trigonometric functions forms a Fourier series, a technique widely used in physics and engineering. [nb 14] [80] The underlying vector space is usually the Hilbert L2(0, 2π) space, for which the functions sin mx and cos mx (m an integer) form an orthogonal base. [81] The Fourier expansion of an L2 f function is a 0 2 + ∑ m = 1 ∞ [a m cos ⁡ (m x) + b m sin ⁡ (m x)] . {\displaystyle {\frac {a_{0}}{2}}+\sum _{m=1}^{\infty }\left[a_{m}\cos \left(mx\right)+b_{m}\sin \left(mx\right)\right].} The am and bm coefficients are called Fourier coefficients of f, and are calculated by formulas[82] to m = 1 π ∫ 0 2 π f (t) cos ⁡ (m t) d t {\displaystyle a_{m}={\frac {1}{\pi }}\int _{0}^{2\pi }f(t)\cos(mt)\,dt} , b m = 1 π ∫ 0 2 π f (t) sin ⁡ (m t) d t . {\displaystyle b_{m}={\frac {1}{\pi }}\int _{0}^{2\pi }f(t)\sin(mt)\,dt.} In physical terms, the function is represented as a sine wave overlay and the coefficients give information about the frequency spectrum of the function. [83] A complex number shape of the Fourier series is also commonly used. [82] The above concrete formulas are consequences of a more general mathematical duality called pontryagin duality. [84] Applied to group R, it produces the classic fourier transformation; an application in physics are reciprocal lattices, where the underlying group is a finite-dimensional real vector space endowed with an additional dotum of a network encoding positions of atoms into crystals. [85] The Fourier series are used to solve threshold value problems in partial differential equations. [86] In 1822, Fourier first used this technique to solve the heat equation. [87] A discrete version of the Fourier series can be used in sampling applications where the function value is known only in a finite number of equally spaced points. In this case, the Fourier series is finite and its value is equal to the values sampled at all points. [88] The set of coefficients is known as the discrete fourier transformation (FtD) of the given sample sequence. DFT is one of the leading digital signal processing tools, a field whose applications include radar, speech coding, image compression. [89] The JPEG image format is an application of the discrete transformation of closely related cosine. [90] The rapid fourier transformation is an algorithm for quickly calculating the discrete fourier transformation. [91] It is used not only to calculate the fourier coefficients, but, using the convolution theorem, also to calculate the convolution of two finite sequences. [92] They, in turn, are applied in digital filters[93] and as a rapid multiplication algorithm for polynomials and large integers (Schönhage-Strassen algorithm). [94] Differential geometry Main article: Tangent space Tangent space for 2 spheres at some point is infinite infinity touching the sphere at this point. The tangent plane to a surface at a point is naturally a vector space whose origin is identified with the point of contact. The tangent plane is the best linear approximation, or linearization, of a surface at a point. [nb 15] Even in a three-dimensional Euclidean space, there is usually no natural way to prescribe a tangent plane base, and so it is conceived as an abstract vector space rather than a real coordinate space. Tangent space is the generalization of different collectors of larger dimensions. [96] Riemanian collectors are collectors whose tangent spaces are endowed with an adequate internal product. [97] Derived from this, Riemann's curvature tensor encodes all the curvatures of a collector into an object, which finds applications in general relativity, for example, where Einstein's curvature tensor describes the matter and energetic content of space-time. [99] The tangent space of a Lie group can naturally be given to the structure of a lie algebra and can be used to classify compact Groups of Lie. [100] Generalizations Vectors packages Main articles: Vector Pack and Tangent A möbius strip package. Locally, it seems that U × R. A beam of vectors is a family of vector spaces parameterized continuously by a topological space X.[96] More precisely, a beam of vectors over X is a topological space E equipped with a continuous map π : And → X in such a way that for each x in X, the fiber π−1(x) is a vector space. The dim box V = 1 is called the line packet. For any Vector Space V, the X × V → X projection causes the X × V product into a trivial vector beam. The vector beams over X are required to be locally a product of X and some vector space (fixed) V: for each x in X, there is a u of x of neighborhood in such a way that the restriction from π to π-1(U) is isomorphic[nb 16] for the trivial package U × V → U. Despite its locally trivial character , vector beams can (depending on the shape of the underlying X space) be distorted in the large (that is, the packet does not have to be (globally isomorphic for) the trivial X × V packet). For example, the Möbius strip can be seen as a line beam over circle S1 (identifying open intervals with the actual line). It is, however, different from the S1 × R cylinder, as the second is advisory, while the former is not.[101] Properties of certain vector beams provide information about the underlying topological space. For example, the tangent package consists of the collection of tangent spaces parameterized by points of a different variety. The tangent beam of circle S1 is globally isomorphic for S1 × R, since there is a global field of non-zero vectors in S1. [nb 17] In contrast, by the naked ball theorem, there is no vector field (tangent) in 2-ball S2 that is everywhere non-zero. [102] K theory studies the classes of of all the vector beams over some topological space. [103] In addition to deepening topical and geometric vision, it has purely algebraic algebraic as the classification of finite-dimensional real division algebras: R, C, quaternions H and octonions O. The cotangent package of a different collector consists, at each point of the collector, of the concurrent space pair. Sections of this package are known as unique shape differentials. Modules Main article: Modules are for rings which vector spaces are for fields: the same axiomas, applied to an R ring instead of an F field, yield modules. [104] The theory of modules, compared to that of vector spaces, is complicated by the presence of ring elements that do not have multiplicative inverse. For example, modules do not need to have bases, as shown in module Z (i.e., abelian group) Z/2Z; the modules they do (including all vector spaces) are known as free modules. However, a vector space can be compactly defined as a module over a ring that is a field, with elements being called vectors. Some authors use the term vector space to signify modules over a split ring. [105] The algebro-geometric interpretation of the commutative rings through their spectrum allows the development of concepts such as local free modules, the algebraic counterpart to vector beams. Affine and projective spaces Main articles: Sharp space and projective space An affine plane (light blue) in R3. It is a two-dimensional subspace displaced by an x vector (red). Approximately, fine spaces are vector spaces whose origins are not specified. [106] More precisely, an amending space is a set with a free transitive vector space action. In particular, a vector space is an affine space about itself, by map V × V → V, (v, a) ↦ a + v. If W is a vector space, then an affine subspace is a subset of W obtained by translating a linear subspace V by a fixed vector x ∈ W; this space is denoted by x + V (it is a coset of V in W) and consists of all vectors of the form x + v to v ∈ V. An important example is the solution space of a system of homogeneous linear equations Ax = b generalizing the homogeneous case b = 0 above. [clarification needed] [107] The space of the solutions is the subspace app x + V where x is a particular solution of the equation, and V is the solution space of the homogeneous equation (the null space of A). The set of one-dimensional subspaces of a fixed finite-dimensional vector space V is known as projective space; it can be used to formalize the idea of parallel lines crossing into infinity. [108] Grassmannians and flag collectors generalize this by parameterizing fixed-dimension k linear subspaces and subspace flags, respectively. See also Vector (mathematics and physics), for a list of various types of Vectors Cartesian coordinate vector system metric space P-vector Space Riesz-Fischer space theorem (mathematics) Vector space ordered Notes ^ It is also common, especially in denote vectors with an arrow on top: v→. ^ This axiom and the next refer to two different operations: scalar multiplication: bv; bv; field multiplication: ab. They do not claim the associativity of any of the operations. More formally, scalar multiplication is a monoide action of the multiplicative monoide of field F in vector space V. ^ Some authors (such as Brown 1991) restrict attention to the R or C fields, but most of the theory is unchanged for an arbitrary field. ^ The indicator functions of the intervals (of which there are infinitely many) are linearly independent, for example. ^ Nomenclature derives from German eigen, which means itself or itself. ^ See also the jordan-Chevalley decomposition. ^ This is typically the case when a vector space is also considered as a like space. In this case, a linear subspace contains the zero vector, while a related subspace does not necessarily contain it. ^ Some authors (like Romano 2005) choose to start with that equivalence ratio and derive the concrete form of V/W from that. ^ That requirement implies that topology gives rise to a uniform structure, Bourbaki 1989, ch. II ^ Triangle inequality for ‖ f + g ‖ p ≤ ‖ ‖ p + ‖ g ‖ p {\displaystyle \left\Vert {f+g}\right\Vert _{p}\leq \left\Vert {f}\right\Vert _{p}\left\Vert\Vert _{p}} is provided by inequality For technical reasons, in the context of functions it is necessary to identify functions that agree almost everywhere to obtain a standard, and not just a seminorm. ^ Many functions in L 2 {\displaystyle L^{2}} of the lebesgue measurement, being unlimited, cannot be integrated with the classic Integral Riemann. Thus, the spaces of Riemann's integral functions would not be complete in l 2 {\displaystyle L^{2}} and orthogonal decomposition would not apply to them. This shows one of the advantages of lebesgue integration., Dudley 1989, §5.3, p. 125 ^ For p ≠2, Lp(Ω) is not a Hilbert space. ^ A base of a Hilbert space is not the same thing as a base in the sense of linear algebra above. [clarification needed] For distinction, the latter is then called hamel base. ^ Although the Fourier series is periodic, the technique can be applied to any L2 function at a range, whereas the function must be continued periodically out of range. See Kreyszig 1988, p. 601 ^ Ie (BSE-3 2001) harv error: no target: CITEREFBSE-32001 (help), the plane passing through the contact point P in such a way that the distance from a Point P1 on the surface to the plane is infinitely small compared to the distance from P1 to P at the limit as P1 approaches P. ^ That is, there is a homeomorphism from π-1(U) to V × U that is restricted to linear isomorphisms between fibers. ^ A line packet, as the Tangent Package of S1 is trivial if and only if there is a section that does not disappear anywhere, see Husemoller 1994, Corollary 8.3. The sections of the tangent beam are vector fields only. Quotes ^ a b Comprehensive List of Algebra Symbols. Math vault. 2020-03-25. 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VI.6 ^ Halmos 1974, p. 28 , Ex. 9 ^ Lang 1987 , Teorema IV.2.1, p. 95 ^ Romano 2005, Th. 2.5 e 2.6, p. 49 ^ Lang 1987, ch. V.1 ^ Lang 1987, ch. V.3., Corollary, p. 106 ^ Lang 1987, Theorem VII.9.8, p. 198 ^ Romano 2005, ch. 8, p. 135-156 ^ Lang 1987, ch. IX.4 ^ Roman 2005, ch. 8, p. 140. ^ Romano 2005, ch. 1, p. 29 ^ Romano 2005, ch. 1, p. 35 ^ Romano 2005, ch. 3, p. 64 ^ Lang 1987, ch. IV.3. ^ Romano 2005, ch. 2, p. 48 ^ Mac Lane 1998 ^ Romano 2005, ch. 1, pp. 31-32 ^ Lang 2002, ch. XVI.1 ^ Romano 2005, Th. 14.3. Veja também Yoneda lemma. ^ Schaefer & Wolff 1999, pp. 204-205 ^ Bourbaki 2004, ch. 2, p. 48 ^ Romano 2005, ch. 9 ^ Naber 2003, ch. 1.2 ^ Treves 1967 ^ Bourbaki 1987 ^ Kreyszig 1989, §4.11-5 ^ Kreyszig 1989, §1.5-5 Choque ^t 1966, Proposição III.7.2 ^ Treves 1967, p. 34-36 ^ Lang 1983, Cor. 4.1.2, p. 69 ^ Treves 1967, ch. 11 ^ Treves 1967, Teorema 11.2, p. 102 ^ Evans 1998, ch. 5 ^ Treves 1967 , ch. 12 ^ Dennery & Krzywicki 1996, p.190 ^ Lang 1993, Th. XIII.6, p. 349 ^ Lang 1993, Th. III.1.1 ^ Choquet 1966, Lemma III.16.11 ^ Kreyszig 1999, Capítulo 11 ^ Griffiths 1995, Capítulo 1 ^ Lang 1993, ch. XVII.3 ^ Lang 2002, ch. III.1, p. 121 ^ Eisenbud 1995, ch. 1.6 ^ Varadarajan 1974 ^ Lang 2002, ch. XVI.7 ^ Lang 2002, ch. XVI.8 ^ Luenberger 1997, §7.13 ^ Ver teoria de representação e representação do grupo. ^ Lang 1993, Ch. XI.1 ^ Evans 1998, Th. 6.2.1 ^ Folland 1992, p. 349 ff ^ Gasquet & Witomski 1999, p. 150 ^ a b Gasquet & Witomski 1999, §4.5 ^ Gasquet & Witomski 1999, p. 57 ^ Loomis 1953, Ch. VII ^ Ashcroft & Mermin 1976, Ch. 5 ^ Kreyszig 1988, p. 667 ^ Fourier 1822 ^ Gasquet & Witomski 1999, p. 67 ^ Ifeachor & Jervis 2001, pp. 3-4, 11 ^ Wallace 1992 ^ Ifeachor & Jervis 2001 , p. 132 ^ Gasquet & Witomski 1999, §10.2 ^ Ifeachor & Jervis 2001, pp. 307-310 ^ Gasquet & Witomski 1999, §10.3 ^ Schönhage & Strassen 1971 ^ a b Spivak 1999, ch. 3 ^ Jost 2005. 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1269324. OCLC 36131259. External links The Linear Wikibook Algebra has a page on the theme: Real Vector Spaces The Linear Wikibook Algebra has a page on the theme: Vector Spaces Vector Space, Encyclopedia of Mathematics, EMS Press, 2001 [1994] Retrieved from 2geometrical object that is the surface of a ball This article is about the concept in three-dimensional geometry. For other uses, see Sphere (disambiguation). Globose redirects here. For the neuroanatomical structure, refer to the Globose nucleus. A two-dimensional perspective projection of a sphere A (from the Greek σφαιρα-sphaira, globe, ball[1]) is a geometric object in three-dimensional space that is the surface of a ball (viz., analogous to circular objects in two dimensions, where a circle circumscribes its disk). As a circle in a two-dimensional space, a sphere is mathematically defined as the set of points that are all at the same distance r from a given point in a three-dimensional space. [2] This distance r is the radius of the ball, which is composed of all points with a distance less than (or, for a closed ball, less than or equal to) r from the given point, which is the center of the mathematical ball. These are also referred to as the radius and center of the sphere, respectively. The longest straight line segment through the ball, connecting two points of the ball, passes through the center and its length is therefore twice the radius; is a diameter of both the ball and its ball. While outside of mathematics the terms sphere and ball are sometimes used interchangeably, in mathematics the above distinction is made between a sphere, which is a two-dimensional closed surface embedded in a three-dimensional Euclidean space, and a ball, which is a three-dimensional shape that includes the sphere and everything inside the sphere (a closed ball), or, more often, only the points inside, but not in the sphere (an open ball). The distinction between ball and ball has not always been maintained and especially older mathematical references speak about a sphere as solid. This is analogous to no plano, onde os termos círculo e disco também podem ser confundidos. Equações em espaço tridimensional Dois raios ortogonais de uma esfera Veja também: função trigonométrica e coordenadas esféricas Na geometria analítica, uma esfera com centro (x0, y0, z0) e raio r é o lócus de todos os pontos (x, y, z) de tal forma que (x − x 0) 2 + (y − y) 2 + (z − z) 2 = r 2 . {\displaystyle (x-x_{0})^{2}+(y-y_{0})^{2}+(z-z_{0})^{2}=r^{2}.} Deixe a, b, c, d, e ser números reais com um ≠ 0 e colocar x 0 = − b a , y 0 = − c a , z 0 = − d a , ρ = b 2 + c 2 + d 2 − a e a 2 . {\displaystyle x_{0}={\frac {-b}{a}},\quad y_{0}={\frac {-c}{a}},\quad z_{0}={\frac {-d}{a}},\quad \rho ={\frac {b^{2}+c^{2}+d^{2}-a}{2} a^ Em seguida, a equação f (x , y , z) = a (x 2 + y 2 + z 2) + 2 (b x + c y + d z) + e = 0 {\displaystyle f(x,y,z)=a(x^{2}+y^{2}+z^{2})+2(bx+cy+dz)+e=2 0} não tem pontos reais como soluções se ρ < 0= {\displaystyle= \rho=> <0} and= is= called= the= equation= of= an= imaginary= sphere.= if= ρ=0 {\displaystyle= \rho=0} ,= the= only= solution= of= f= (= x= ,= y= ,= z=)=0 {\displaystyle= f(x,y,z)=0} is= the= point= p= 0=(x= 0= ,= y= 0= ,= z= 0=)= {\displaystyle= p_{0}=(x_{0},y_{0},z_{0})} and= the= equation= is= said= to= be= the= equation= of= a= point= sphere.= finally ,= in= the= case= ρ=>0 {\displaystyle \rho >0} , f (x , y , z) = 0 {\displaystyle f(x,y,z)=0} é uma equação de uma esfera cujo centro é P 0 {\displaystyle P_{0}} e cujo raio é ρ {\displaystyle {\sqrt {\\sqrt {\rho}} 2] Se uma na equação acima é zero, então f(x, y, z) = 0 é a equação de um plano. Assim, um plano pode ser pensado como uma esfera de raio infinito cujo centro é um ponto no infinito. [3] Os pontos na esfera com raio r > 0 {\displaystyle r>0} e centro (x 0 , y 0 , z 0) {\displaystyle (x_{0},y_{0},z_{0})} pode ser parametrizado via x = x 0 + r sin φ cos φ y = y 0 + r sin < 2= π=)= z=z 0= += r= cos= = θ= {\displaystyle= {\begin{aligned}x&=x_{0}+r\sin \theta= \;\cos= \varphi= \\y&=y_{0}+r\sin \theta= \;\sin= \varphi= \qquad= (0\leq= \theta= \ leq= \pi= ,\;0\leq= \varphi=> <2\pi)\\z&=z_{0}+r\cos \theta \,\end{aligned}}} [4] The parameter θ {\displaystyle \theta } can be associated with the angle counted positive from the direction of the positive z-axis through the center to the radius-vector, and the parameter φ {\displaystyle \varphi } can be associated with the angle counted positive from the direction of the positive x-axis through the center to the projection of the radius-vector on the xy-plane. A sphere of any radius centered at zero is an integral surface of the following differential form: x d x + y d y + z d z = 0. {\displaystyle x\,dx+y\,dy+z\,dz=0.} This equation reflects that position and velocity vectors of a point, (x, y, z) and (dx, dy, dz), traveling on the sphere are always orthogonal to each other. A sphere can also be constructed as the surface formed by rotating a circle about any of its diameters. a circle is a special type of ellipse, a sphere is a special type of)\\z&=z_{0}+r\cos \theta= \,\end{aligned}}}= [4]= the= parameter= θ= {\displaystyle= \theta= }= can= be= associated= with= the= angle= counted= positive= from= the= direction= of= the= positive= z-axis= through= the= center= to= the= radius-vector,= and= the= parameter= φ= {\displaystyle= \varphi= }= can= be= associated= with= the= angle= counted= positive= from= the= direction= of= the= positive= x-axis= through= the= center= to= the= projection= of= the= radius-vector= on= the= xy-plane.= a= sphere= of= any= radius= centered= at= zero= is= an= integral= surface= of= the= following= differential= form:= x= d= x= += y= d= y= += z= d= z=0. {\displaystyle= x\,dx+y\,dy+z\,dz=0.} this= equation= reflects= that= position= and= velocity= vectors= of= a= point,= (x,= y,= z)= and= (dx,= dy,= dz),= traveling= on= the= sphere= are= always= orthogonal= to= each= other.= a= sphere= can= also= be= constructed= as= the= surface= formed= by= rotating= a= circle= about= any= of= its= diameters.= since= a= circle= is= a= special= type= of= ellipse,= a= sphere= is= a= special= type= of=></2\pi)\\z&=z_{0}+r\cos \theta \,\end{aligned}}} [4] The parameter θ {\displaystyle \theta } can be associated with the angle counted positive from the direction of the positive z-axis through the center to the radius-vector, and the parameter φ {\displaystyle \varphi } can be associated with the angle counted positive from the direction of the positive x-axis through the center to the projection of the radius-vector on the xy-plane. A sphere of any radius centered at zero is an integral surface of the following differential form: x d x + y d y + z d z = 0. {\displaystyle x\,dx+y\,dy+z\,dz=0.} This equation reflects that position and velocity vectors of a point, (x, y, z) and (dx, dy, dz), traveling on the sphere are always orthogonal to each other. A sphere can also be constructed as the surface formed by rotating a circle about any of its diameters. Since a circle is a special type of ellipse, a sphere is a special type of > sin φ (0 ≤ φ ≤ π , 0 ≤ φ</0}> φ</0}> of the revolution. Replacing the circle with an ellipse rotated on its main axis, the shape becomes a prolate ballpoint; rotated on the smaller shaft, an oblate ballpoint. [5] Closed volume sphere and circumscribed cylinder In three dimensions, the volume within a sphere (i.e., the volume of a ball, but classically referred to as the volume of a sphere) is V = 4 3 π r 3 = π 6 d 3 ≈ 0.5236 - d 3 {\displaystyle V={\frac {4}{3}}\pi r^{3}={\frac {\pi }{6}}\ d^{3}\approximately 0.5236\cdot d^{3}} where the radius is and d is the diameter of the sphere. Archimedes first derived this formula, showing that the volume within a sphere is twice the volume between the sphere and the circumscribed cylinder of that sphere (having the height and diameter equal to the diameter of the sphere). [6] This can be proven by by inserting an upside-down cone into the semisphere, noting that the area of a cone cross section plus the area of a cross section of the ball is the same as the cross-section area of the circumscribed cylinder, and applying the Cavalieri principle. [7] This formula can also be derived using integral calculus, i.e. disk integration to sum the volumes of an infinitely small circular disk of infinitely small thickness stacked side by side and centered along the x-axis of x = −r to x = r, assuming that the r-ray sphere is centered on the origin. In any x, the incremental volume (δV) is equal to the product of the cross-sectional area of the disk in x and its thickness (δx): δ V ≈ π and 2 - δ x . {\displaystyle \delta V\approx \pi y^{2}\cdot \delta x.} The total volume is the sum of all incremental volumes: V ≈ ∑ π and 2 - δ x . {\displaystyle V\approx \sum \pi y^{2}\cdot \delta x.} At the limit as δx approaches zero,[8] this equation becomes: V = ∫ − r r π and 2 d x . {\displaystyle V=\int _{-r}^{r}\pi y^{2}dx.} In any x, a right-angle triangle connects x, y, and r to the source; therefore applying the pythagorean theorem yields: y 2 = r 2 − x 2 . {\displaystyle y^{2}=r^{2}-x^{2}.} Using this substitution gives V = ∫ − r r π (r 2 − x 2) d , {\displaystyle V=\int _{-r}^{r}{pi \left(r^{2}-x^{2}\right)dx,} which can be evaluated to give the result V = π [r 2 x − x 3 3] − r = π (r 3 − 3 3) − π (− r 3 + r 3 3) = 4 3 π 3 . {\displaystyle V=\pi \left[r^{2}x-{\frac {x^{3}}{3}}}*right]_{-r}^{r}='s left(r^{3}-{\frac {r^{3}}{3}}\right)-\pi \left(-r^{3}+{\frac {r^{3}}{3}}\right)={{frac/frac {4}{3}}}\pi r^{3}.} An alternative formula is found using spherical coordinates, with volume element d V = r 2 sin φ d d φ d φ {\displaystyle dV=r^{2}\sin \theta \,dr\,d\theta \,d\varphi } so V = ∫ 0 2 π ∫ 0 π ∫ 0 r ' 2 sin φ d r ' d φ d φ = 2 π ∫ 0 π ∫ 0 r ' 2 sin φ d ' d φ = 4 π ∫ 0 r r ' 2 d r ' = 4 3 π r 3 . {\displaystyle V=\int _{0}^{2\pi }\int _{0}^{\pi }\int _{0}^{r}r'^{2}\sin \theta \,dr'\,d\theta \,d\varphi =2\pi \int _{0}^{\pi }\int _{0}^{r}r'^{2}\sin \theta \,dr'\,d\theta =4\pi \int r^{3}.} For more practical purposes, the volume within a sphere inscribed in a cube can be approximated as 52.4% of the cube volume, since V = π/6 d3, where d is the diameter of the sphere and also the length of one side of the cube and π/6 ≈ 0.5236. For example, a sphere with a diameter of 1 m has 52.4% of the volume of a cube with an edge length of 1 m, or about 0.524 m3. Surface area The surface area of a radius sphere r is: A = 4 π r 2 . {\displaystyle A=4\pi r^{2}.} Archimedes first derived this formula[9] from the fact that the projection to the lateral surface of a circumscribed cylinder is preservation of the area. [10] Another approach to obtaining the formula comes from the fact that it is equal to the formula derivative for volume relative to r because the total volume within an r-ray sphere can be thought of as the sum of the surface area of an infinitesimal spherical shell concentrically stacked within each other from radius 0 to radius r. In infinitesimal thickness the discrepancy between the inner and outer surface area of any shell is infinitesimal, and the elemental volume in radius r is simply the product of the surface area in the r radius and infinitesimal thickness. In any radius r,[note 1] the incremental volume (δV) is equal to the surface product in radius r (R(r)) and the thickness of a shell (δr): δ V ≈ A (r) - δ r . {\displaystyle \delta V\approx. A(r)\cdot \delta r.} The total volume is the sum of all shell volumes: V ≈ ∑ A (r) (δ r . {\displaystyle V\approx \sum A(r)\cdot \delta r.} At the limit as δr approaches zero[8] this equation becomes: V = ∫ 0 r A (r) d r . {\displaystyle V=\int _{0}^{r}A(r)\,dr.} Substitute V: 4 3 π r 3 = ∫ 0 r A (r) d r . {\displaystyle {\frac {4}{3}}\pi r^{3}=\int _{0}^{r}A(r)\,dr.} Differentiating both sides of this equation from r yields A as a function of r: 4 π r 2 = A (r). {\displaystyle 4\pi r^{2}=A(r).} This is usually abbreviated as: A = 4 π r 2 , {\displaystyle A=4\pi r^{2},} where they are now considered as the fixed radius of the sphere. Alternatively, the area element in the sphere is given in spherical coordinates per dA = r2 sin φ dφ dφ. In Cartesian coordinates, the area element is [citation required] d S = r r 2 − ∑ i ≠ k x i 2 ∏ i ≠ k d x i , ∀ k . {\displaystyle dS={\frac {r}{\sqrt {r^{2}-{\displaystyle \sum _{ieq k}x_{i}^{2}}\prod _{ieq k}dx_{i},\;\forall k.} Thus, the total area can be obtained by integration: A = ∫ 0 2 π ∫ 0 π r 2 sin φ d φ d φ = 4 π r 2 . {\displaystyle A=\int _{0}^{2\pi }\int _{0}^{\pi }r^{2}\sin \theta \,d\theta \,d\varphi =4\pi r^{2}.} The sphere has the smallest surface area of all surfaces that include a given volume, and includes the largest volume among all closed surfaces with a given surface area. [11] The sphere therefore appears in nature: for example, bubbles and small drops of water are approximately spherical because the surface tension minimizes minimizes Area. The surface area relative to the mass of a ball is called the specific surface area and can be expressed from the equations indicated above as S A = A V ρ = 3 r ρ , {\displaystyle \mathrm {SSA} ={\frac {A}{V\rho }}={{\frac {3}{r\rho }} where ρ is the density (the mass ratio for the volume). Curves in a sphere Flat section of a sphere: 1 circle coaxial crossing of a sphere and a cylinder: 2 circles Circles The intersection of a sphere and a plane is a circle, a point, or an empty one. In the case of a circle, the circle can describe by a parametric equation x → = (and → 0 + and → 1 cos t + and → 2 sin t) T {\displaystyle \;{ \vec {x}}=({\ve c {e}}_{0}+{\vec {e}}_{1}\cos t+{\vec {e}}_{2}\sin t)^{T}\;} : see plane section of an ellipsoid. But more complicated surfaces can intersect a sphere in circles, too: a non-empty intersection of a sphere with a revolution surface, whose axis contains the center of the sphere (they are coaxial) consists of circles and/or points. The diagram shows the case, where the intersection of a cylinder and a sphere consists of two circles. If the radius of the cylinder were equal to the radius of the sphere, the intersection would be a circle, where both surfaces are tangent. In the case of a spartad with the same center and main axis of the sphere, the intersection would consist of two points (vertices), where the surfaces are tangent. Clelia spherical spiral curve with c = 8 {\displaystyle c=8} If the sphere is described by a parametric representation x → = (r cos φ cos φ , r cos φ sin φ , r sin φ) T {\displaystyle {\vec {x}}=(r\cos \theta \cos \varphi,r\cos \theta \sin \varphi,r\sin \theta)^{T}} one gets clelia's curves if the angles are connected by equation φ = c φ , c > 0 . {\displaystyle \varphi =c\;\theta \;,\ c>0\;.} Special cases are: Viviani curve (c = 1 {\displaystyle c=1}) and spherical spirals (c > 2 {\displaystyle c>2}). Loxodrome Loxodrome In navigation, a line of rhumb or loxodrome is an arc that crosses all meridians of longitude at the same angle. A snuff line is not a spherical spiral. There is no simple connection between the angles φ {\displaystyle \varphi } and φ {\displaystyle \theta } . Intersection of a sphere with a more general surface crossing sphere-cylinder If a sphere is interseried by another surface, there may be more complicated spherical curves. Example: sphere - cylinder The intersection of the sphere with equation x 2 + y 2 + 2 + z 2 = r 2 {\displaystyle \;x^{2}+y^{2}+z^{2}=r^{2}\;} and the cylinder with equation (y − y 0) 2 + z 2 = a 2 , y 0 ≠ 0 {\displaystyle \;(y-y_{0})^{2}+z^{2}=a^{2},\;y_{0}eq 0\;} it's not just one or two circles. It is the solution of the nonlinear system of equations x 2 + y 2 + z 2 − r 2 = 0 {\displaystyle x^{2}+y^{2}+z^{2}-r^{2}=0} (y − y 0) 2 + z 2 − to 2 = 0 . {\displaystyle (y-y_{0})^{2}+z^{2}-a^{2}=0\ .} (see implicit curve and diagram) Geometric properties A sphere is determined exclusively four points that are not coplanar. More More a sphere is determined exclusively by four conditions, such as passing through a point, being tangent to a plane, etc.[12] This property is analogous to the property that three non-collinear points determine a single circle in a plane. Consequently, a sphere is determined exclusively by (that is, passes through) a circle and a point that is not in the plane of that circle. Examining the common solutions of the equations of two spheres, one can see that two spheres intersect in a circle and the plane containing this circle is called the radical plane of the intersectoral spheres. [13] Although the radical plane is a real plane, the circle can be imaginary (the spheres have no real point in common) or consist of a single point (the spheres are tangent at that point). [14] The angle between two spheres at a real point of intersection is the dihedral angle determined by the tangent planes to the spheres at that point. Two spheres intersect at the same angle at all points of their intersecting circle. [15] They intersect at right angles (they are orthogonal) if and only if the square of the distance between their centers is equal to the sum of the squares of their rays. [3] Ball pencilS Main article: Pencil (mathematics) § Ball pencilS If f(x, y, z) = 0 and g(x, y, z) = 0 are the equations of two distinct spheres s s f (x , y , z) + t g (x , y , z) = 0 {\displaystyle sf(x,y,z)+tg(x,y,z)=0} is also the equation of a sphere for arbitrary values of parameters s and t. The set of all spheres that satisfy this equation is called an equation is called a sphere for arbitrary values of parameters s and t. The set of all spheres that satisfy this equation is called an equation is called a sphere is called a sphere for arbitrary values of parameters s and t. The set of all spheres that satisfy this equation is called an equation is called a sphere is called a sphere for arbitrary values of parameters s and t. The set of all spheres that satisfy this equation is called an equation is called a sphere is called a sphere pencil sowers determined by the two original spheres. In this definition, a sphere is allowed to be a plane (infinite radius, center at infinity) and if both original spheres are flat, then all pencil spheres are flat, otherwise there is only one plane (the radical plane) in the pencil. [3] Terminology plane sections A large circle in the sphere has the same center and radius of the sphere —consequently dividing it into two equal parts. The plane sections of a sphere are called spherical sections—which are great circles for airplanes through the center of the sphere or small circles for everyone else. [16] Any plane that includes the center of a sphere divides it into two equal hemispheres. Any two intersectional planes that include the center of a sphere subdivide the sphere into four lunes or biangles, the vertices of which coincide with the antipodal points lying on the intersection line of the aircraft. Branches of geometry Non-Euclidean Distance Any pair of points on a sphere that lays in a straight line through the center of the sphere (i.e. the diameter) are called antipodal points — in the sphere, the distance between them is exactly half the length of the circumference. [note 2] Any other pair (i.e. non-antipodal) of distinct points in a sphere resides in a large single circle, segmenting it into a smaller arc (i.e. smaller) and a larger (i.e. longer), and having the of the smaller arc being the shortest distance between on the sphere. [note 3] Spherical geometry[note 4] shares many properties analogous to the Euclidean once equipped with this large circle distance. Differential geometry And a much more abstract generalization of geometry also uses the same concept of distance in the Riemannian circle. The hemisphere is conjecture to be the ideal isometric filling (less area) of the Riemanian circle. Projective geometry The antipodeal quotient of the sphere is the surface called the actual projective plane, which can also be considered as the northern hemisphere with antipodeal points of the equator identified. Terms of geography borrowed directly from the geography of the Earth, although their spheroidal form has larger or smaller departures from a perfect sphere (see geoide), are widely well understood. In geometry not related to astronomical bodies, geocentric terminology should be used only for illustration and annotated as such, unless there is no chance of misunderstanding. Poles, longitude, and latitudes If a particular point in a sphere is (arbitrarily) designated as its north pole, its antipodal point is called the south pole. The large equidistant circle for each is then the equator. Large circles across the poles are called longitude lines (or meridians). A line not on the sphere, but through its center connecting the two poles can be called the axis of rotation. Circles in the parallel sphere (i.e. not large circles) to the equator are latitude lines. Major Generalizations articles: n-sphere and Metric space Dimensionality Spheres can be generalized to spaces of any number of dimensions. For any natural number n, an n-sphere, often written as Sn, is the set of points in the (n + 1)-dimensional Euclidean space that are at a fixed distance r from a central point of that space, where they are, as before, a positive real number. In particular: S0: a sphere 0 is a pair of endpoints of a range [−r, r] of the actual line S1: a sphere of 1 sphere is a radius circle r S2: a sphere of 2 spheres is a common sphere S3: a sphere of 3 spheres is a sphere in 4-dimensional Euclidean space. Spheres for n > 2 are sometimes called hyperspheres. The n sphere of the unit radius centered on the origin is denoted Sn and is often referred to as the n-sphere. Note that the common sphere is a 2-sphere sphere, because it is a two-dimensional surface (which is embedded in three-dimensional space). The surface area of the unit (n-1)-sphere is 2 π n 2 Γ (n 2) {\displaystyle {\frac {2\pi ^{\frac {n}{2}}}{\Gamma \left({\frac {n}{2}}\right)} where Γ(z) is euler's gamma function. Another expression for the surface is { (2 π) n / 2 r n − 1 2 • 4 ⋯ (n − 2) , if n is same ; 2 (2 π) (n − 1) / 2 r n − 1 1 • 3 ⋯ (n − 2) , if n is strange . {\displaystyle {\begin{cases}\displaystyle {\frac {(2\pi)^{n/2}\,r^{n-1}}{2\cdot 4\cdots (n-2)}},&{\text{if }}n{\text is even}};\displaystyle {2(2\pi)^{(n-1)/2}\,r^{n-1}} {1\cdot 3\cdots (n-2)}},&{\text{if }}n{\text{ is odd}}.\end{cases}}} and the volume is the surface area r/n or { (2 π) n / 2 r n 2 • 4 ⋯ n , if n is even ; 2 (2 π) (n − 1) / 2 r n 1 • 3 ⋯ n , if n is strange . {\displaystyle {\begin{cases}\displaystyle {\frac {(2\pi)^{n/2}\,r^{n}}{2\cdot 4\cdots n}},&{\text{if }}n{\text is even}};\display {\frac {2(2\pi)^{(n-1)/2}\,r^{n}}{1\cdot 3\cdots n}},&{\text{if }}n{\text{ is odd}}.\end{cases}}} General recursive formulas also exist for the volume of an n-ball. Metric spaces More generally, in a metric space (E,d), the sphere of the center x and radius r > 0 is the set of points y such that d(x,y) = r. If the center is a distinct point that is considered the origin of e, as in a standard space, it is not mentioned in the definition and notation. The same applies to the radius if it is taken to equal one, as in the case of a unit sphere. Unlike a ball, even a large ball can be an empty set. For example, in Zn with Euclidean metric, a sphere of radius r is zero if r2 can be written as sum of n squares of integers. Topology In topology, an n-sphere is defined as a homeomorphic space up to the limit of one (n + 1)-ball; thus, it is homeomorphic to the Euclidean n-sphere, but perhaps without its metric. A sphere 0 is a pair of points with the discrete topology. A sphere of 1 is a circle (up to homeomorphism); so, for example, (the image of) any node is a sphere of 1. A 2-ball sphere is a common sphere (up to homeomorphism); so, for example, any ball point is a sphere of 2 spheres. The n-sphere is denoted Sn. It is an example of a compact topological collector without limits. A sphere does not need to be soft; if it is soft, it does not need to be diffeomorphic to the Euclidean sphere (an exotic sphere). Heine-Borel's theorem implies that a Euclidean n-sphere is compact. The sphere is the inverse image of a point defined under continuous function || x||. Therefore, the sphere is closed. Sn is also limited; therefore, it is compact. Notably, it is possible to transform a common sphere inside out into a three-dimensional space with possible self-intersections, but without creating any crease, into a process called sphere eversion. Spherical geometry Large circle in a sphere Main article: Spherical geometry The basic elements of euclidean plane geometry are points and lines. In the sphere, the points are defined in the usual direction. The analogue of the line is the geodesic, which is a large circle; the defining characteristic of a large circle is that the plane containing all its points also passes through the center of the sphere. Measuring by arc length shows that the shortest path between two points lying on the sphere is the shortest segment of the large circle that includes the points. Many theorems of classical geometry are also true to spherical geometry, but not all do so because the sphere does not satisfy some of the postulates of classical geometry, including the parallel postulate. In spherical trigonometry, angles are defined between large Spherical trigonometry differs from common trigonometry many aspects. For example, the sum of the interior angles of a spherical triangle always exceeds 180 degrees. In addition, two similar spherical triangles are congruent. Eleven properties of the sphere A normal vector for a sphere, a normal plane and its normal section. The curvature of the intersection curve is the sectional curvature. For the sphere each normal section through a certain point will be a circle of the same radius: the radius of the sphere. This means that each point in the sphere will be an umbilical point. In their book Geometria and the Imagination[17], David Hilbert and Stephan Cohn-Vossen describe eleven properties of the sphere and discuss whether these properties exclusively determine the sphere. Several properties hold up for the plane, which can be thought of as a sphere with infinite radius. These properties are: The points on the sphere are all the same distance from a fixed point. In addition, the ratio of the distance of your points from two fixed points is constant. The first part is the usual definition of the sphere and determines it exclusively. The second part can be easily deduced and follows a similar result of Perga's Apolnium to the circle. This second part also applies to the plane. The contours and plane sections of the sphere are circles. This property defines the sphere in a unique way. The sphere has constant width and constant waist. The width of a surface is the distance between pairs of parallel tangent planes. Numerous other closed convex surfaces have constant width, for example, the meissner body. The waist of a surface is the circumference of the border of its orthogonal projection on a plane. Each of these properties implies the other. All points of a sphere are navels. At any point on a surface, a normal direction is at right angles to the surface because the sphere these are the lines radiating out of the center of the sphere. The intersection of a plane that contains the normal with the surface will form a curve called the normal section, and the curvature of this curve is the normal curvature. For most points on most surfaces, different sections will have different curvatures; the maximum and minimum values of these are called main curvatures. Any closed surface will have at least four points called umbilical points. In a navel all sectional curvatures are equal; in particular, the main curvatures are the same. Umbilical points can be thought of as points where the surface is closely approximated by a sphere. For the sphere the curvatures of all normal sections are equal, so each point is a navel. The ball and plane are the only surfaces with this property. The sphere does not have a center surface. For a given normal section there is a circle of curvature that is equal to the sectional curvature, is tangent to the surface, and the center lines of which are along the normal line. For example, the two centres maximum and minimum sectional curvatures are focal points, and the set of all these centers forms the focal surface. For most surfaces, the focal surface forms two sheets each a surface and are in umbilical points. Several cases are special: * For channel surfaces one sheet forms a curve and the other sheet is a surface * For cones, cylinders, tori and cichlids both sheets form curves. * For the sphere, the center of each osculante circle is in the center of the sphere and the focal surface forms a single point. This property is unique for the sphere. All geodesics of the sphere are closed curves. Geodesics are curves on a surface that give the shortest distance between two points. They are a generalization of the concept of straight line in the plane. For the sphere, geodesics are large circles. Many other surfaces share this property. Of all solids with a given volume, the sphere is the one with the smallest surface area; of all solids with a given surface area, the sphere has the largest volume. It comes from isoperimetric inequality. These properties define the sphere uniquely and can be seen in soap bubbles: a soap bubble will include a fixed volume, and surface tension minimizes its surface area for that volume. A freely floating soap bubble therefore approximates a sphere (although external forces such as gravity slightly distort the shape of the bubble). It can also be seen on planets and stars where gravity minimizes the surface area for large celestial bodies. The sphere has the lowest total average curvature between all convex solids with a given surface area. The average curvature is the average of the two main curvatures, which is constant because the two main curvatures are constant at all points of the sphere. The sphere has constant average curvature. The sphere is the only embedded surface that lacks limits or singularities with constant positive average curvature. Other immersed surfaces such as minimal surfaces have constant average curvature. The sphere has constant positive Gaussian curvature. Gaussian curvature is the product of the two main curvatures. It is an intrinsic property that can be determined by measuring length and angles and is independent of how the surface is embedded in space. Thus, bending a surface will not alter gaussian curvature, and other surfaces with constant positive Gaussian curvature can be obtained by cutting a small crack in the sphere and bending it. All these other surfaces would have boundaries, and the sphere is the only surface that does not have a boundary with constant and positive Gaussian curvature. The pseudosphere is an example of a surface with constant negative Gaussian curvature. The sphere is transformed into itself by a family of three parameters of rigid movements. Rotating around any axis a unitsphere in the origin will map the sphere over itself. Any over a line through the origin can be expressed as a combination of rotations around the three-coordinate axis (see Euler angles). Euler). there is a family of three rotation parameters in such a way that each rotation transforms the sphere into itself; this family is the SO rotation group(3). The plane is the only other surface with a family of three transformation parameters (translations along the x and y axes and rotations around the origin). Circular cylinders are the only surfaces with families of two parameters of rigid movements and the surfaces of the revolution and helicals are the only surfaces with a family of a parameter. Gallery An image of one of the most precise human spheres, while refracting einstein's image in the background. This sphere was a quartz gyroscope fused for the B gravitational probe experiment, and differs in the form of a perfect sphere by no more than 40 atoms (less than 10 nm) thick. It was announced on July 1, 2008 that Australian scientists had created even more near-perfect spheres, with an accuracy of 0.3 nm, as part of an international hunt to find a new global standard kilogram. [18] Deck of cards illustrating engineering instruments, England, 1702. King of swords: Spheres Regions See also: Ball (mathematics) § Spherical cover Spherical Spherical Sector Spherical Wedge Spherical Spherical Sphere See also 3 spheres Affine Sphere Alexander horned celestial spheres Cube Curvature Directional Statistics Dome (mathematics) Sphere Dyson Hand with Reflecting Sphere, M.C. Escher self-portrait drawing illustrating the reflection and optical properties of a mirrored sphere Sphere Hoberman Sphere Homotology groups homotopy spheres Homotopy Sphere Hypersphere Lenart Sphere Ring problem orb (optic) Pseudosphere Riemann sphere Solid Angle Sphere packing spherical coordinates Terra Spherical Herric, indicatrix tangent of a constant precession curve of spherical spherical sphericity Sphericity Zoll sphere Notes and notes ^ r references is being considered as a variable in this computation. ^ No matter which direction is chosen, the distance is the radius of the sphere × π. ^ The distance between two non-distinct points (i.e., a point and itself) on the sphere is zero. ^ Although not flat, a sphere is two-dimensional as it comprises only the surface of a solid ball. 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